ESTIMATING MASS OF SIGMA-MESON AND STUDY ON APPLICATION OF THE LINEAR SIGMA-MODEL

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Abstract
Whether the $\sigma - meson$ ($f_0(600)$) exists as a real particle is a long-standing problem in both particle physics and nuclear physics. In this work, we analyze the deuteron binding energy in the linear σ model and by fitting the data, we are able to determine the range of m_σ and also investigate applicability of the linear σ model for the interaction between hadrons in the energy region of MeV's. Our result shows that the best fit to the data of the deuteron binding energy and other experimental data about deuteron advocates a narrow range for the $\sigma-$meson mass as $520 \leq m_\sigma \leq 580$ MeV and the concrete values depend on the input parameters such as the couplings. Inversely fitting the experimental data, our results set constraints on the couplings. The other relevant phenomenological parameters in the model are simultaneously obtained.

I. Introduction

Deuteron is the simplest bound state of nucleons, so that a study on it may provide us with information of the nuclear force and the fundamental principle of the strong interaction.

Definitely nucleons are bound together by the strong interaction whose fundamental theory is QCD. Unfortunately, so far, we are only confident with the calculation of perturbative QCD at large momentum transfer i.e. at high energy scales where α_s is sufficiently small due to the asymptotic freedom. By contrast, for the lower energy processes, when one evaluates the effects of hadronization or binding energies, the non-perturbative QCD effects dominate and the traditional perturbative treatments of the field theory are no longer applicable. It turns out that one needs to invoke some phenomenological models in the case. As a matter of fact, most

reasonable models express certain aspects of the real physical world, but not complete. There is a limited range for each model, beyond the limit, application of the model is not legitimate. On other side, for a certain range, all possible models are equivalent for describing the concerned physics. Therefore, in some sense, various models are parallel. This is understood as we use the Cornell potential, the Richardson potential or even the logarithmic potential to describe the bound states of heavy quarks such as J/ψ, Υ etc. and obtain close results with specific parameters. Of course, it by no means manifests that they are real physics, but just serve as an effective model and are applicable to the some concerned processes.

For the quark-bound states, i.e. hadrons, to deal with the non-perturbative QCD effects, we can use, for example, the QCD sum rules, potential models, the bag model, as well as the lattice calculations to evaluate the spectra and other properties. It is believed that even though α_s is large the quark-gluon picture is still valid. By contrast, for the nucleon-nucleon interaction (or hadron-hadron interactions when both of them are in color singlet), even though it is in principle due to QCD, there is no single-gluon exchange between the nucleons because they reside in color-singlet. Instead, just as the familiar Van der Waals force between moleculas in classical physics, which is induced by the electric and magnetic multipoles, the strong interaction between nucleons is the interaction between the chromo-electric and magnetic multipoles of nucleons. So far, there is no a successful way to derive an effective form of such strong interaction at the hadron level from the fundamental QCD theory yet, but on other side, it is also believed that the chiral Lagrangian is consistent with the general principles of QCD and could serve as an effective theory of strong interaction at hadron level [1]. An alternative version of the chiral Lagrangian is the linear sigma model where the $\sigma-$particle stands as an independent resonance. Recently, many theorists and experimentalists are searching for σ which may play an important role in nuclear physics, because the σ meson can provide reasonable middle-range nuclear force. However, the present data [2, 3] allow a rather wide range for both the mass and lifetime of $\sigma-$particle. Thus the existence of σ as a real physical resonance at the hadron spectra of low energy is a long-standing problem in both particle physics and nuclear physics. It appears as $f_0(600)$ in the recent data book, but there still is an acute dispute about its existence. In this work, we hope to gain more information about this resonance through studying the deuteron structure.

Deuteron is a familiar object in nuclear physics for many years, it may be viewed as an ideal lab for probing any possible mechanism of nuclear force. Many authors have recently studied the deuteron structure from different angles[4], and also looked for new physics, such as the three-nucleon force via deuteron breaking up [5]. As indicated above, we trust that some models are equivalent in principle for the energy range of a few MeVs, i.e. we can apply any of them. By fitting data, a set of concerned parameters should be obtained.

Entem and Machleidt [6] developed an accurate NN potential based on the chiral effective theory. Parallel to their work, in this paper we employ the linear sigma model as an alternative approach to study the NN potential. The main difference from [7] is that σ is an independent particle here and we can neglect the two-pion exchange contributions. In realistic physics world σ may be a resonance of certain mass and has the quantum number of two pions in an S-wave. The calculation now is much simpler because we do not need to calculate loop diagrams and deal with the renormalization. In this model, we derive the NN-effective potential and evaluate the spectrum, then compare it with data. In fact, besides the deuteron binding energy, there

are some other parameters which can be experimentally measured, such as the mixing fraction of the s- and d-waves, its mean charge radius etc. Therefore, when we determine the mass of σ meson, we need also to take into account the constraints from the relevant measurements on the parameters.

In fact, similar to the treatment with the chiral Lagrangian, in this work all the coefficients at the effective vertices are derived by fitting data of some physical processes, where all external particles are supposed to be on their mass-shells. Therefore, in our derivation of the effective potential, we need to introduce a reasonable form factor at the effective vertices which manifests the effects of the inner quark-gluon structure of nucleons and partly compensate the off-shell effects of the exchanged mesons at t-channel. Besides we also take into account the contributions of the light vector mesons to the potential.

This model should be tested by evaluating the deuteron binding energy which has been measured with high accuracy and we wish to apply the model along with the obtained parameters to the discussions on the molecular states [8] which are important for understanding the hadron spectra, especially at middle energy ranges.

In this work, we first derive the effective potential between nucleons. The tensor part of the potential results in a mixing between the s-wave and d-waves. Namely it not only contributes to the diagonal elements, but also to the off-diagonal ones of the hamiltonian matrix. Diagonalizing the matrix, we obtain the mass eigenvalues, one of them is the deuteron mass and we can argue that the larger one corresponds to an unstable resonance and does not exist in the physical world.

This paper is organized as follows, after the introduction, we derive the formulation and in Sec.III, we present our numerical results and determine the concerned parameters, finally the last section is devoted to our conclusion and discussion.

II. Formulation

Following the traditional method [9] we derive the effective potential between proton and neutron.

As discussed above we introduce a form factor to compensate the off-shell effects of the exchanged mesons. At each vertex, the form factor is written as[10]

$$\frac{\Lambda^2 - M_m^2}{\Lambda^2 - q^2}, \tag{1}$$

where Λ is a phenomenological parameter and its value is near 1 GeV. It is observed that as $q^2 \to 0$ it becomes a number and if $\Lambda \gg M_m$, it turns to be unity, in this case, the distance is infinitely large and the vertex looks like a perfect point, so the form factor is simply 1 or a constant. Whereas, as $q^2 \to \infty$, the form factor approaches to zero, in this situation, the distance becomes very small, the inner structure (quark, gluon degrees of freedom) would manifest itself and the whole picture of hadron interaction is no longer valid, so the form factor is zero which cuts off the end effects.

To derive an effective potential, one sets $q_0 = 0$ and writes down the elastic scattering amplitude in the momentum space and then carries out a Fourier transformation turning the

amplitude into an effective potential in the configuration space. The best way to order the operators in the derivation is the Weyl ordering scheme [11].

In the linear σ model, the effective Lagrangian is

$$L = g\bar{\psi}(\sigma + \gamma_5 \boldsymbol{\tau} \cdot \boldsymbol{\pi})\psi. \tag{2}$$

(a) Via single-pion exchange.

The effective vertex is

$$\begin{aligned} L &= g\bar{\psi}\gamma_5 \boldsymbol{\tau} \cdot \boldsymbol{\pi}\psi \\ &= \sqrt{2}g\Big[\bar{p}\gamma_5 n\pi^+ + \bar{n}\gamma_5 p\pi^- + \frac{1}{\sqrt{2}}(\bar{p}\gamma_5 p - \bar{n}\gamma_5 n)\pi^0\Big], \end{aligned} \tag{3}$$

where ψ is the wavefunction of nucleon. The scattering amplitude is

$$M = [-2g^2\bar{p}\gamma_5 n\bar{n}\gamma_5 p + g^2\bar{p}\gamma_5 p\bar{n}\gamma_5 n] \cdot \frac{1}{q^2 - m_\pi^2}, \tag{4}$$

it is noted that following the traditional treatment, the wavefunctions of nucleons in the amplitude are that of free nucleons. Setting $q_0 = 0$ and including the aforementioned form factor at the vertices, the potential in the momentum space has a form

$$\begin{aligned} V_\pi(\mathbf{q}) &= \frac{g^2}{4m^2(\mathbf{q}^2 + m_\pi^2)}\Big[4\mathbf{p}_1^2 - 4(\boldsymbol{\sigma}_1 \cdot \mathbf{p}_1)(\boldsymbol{\sigma}_2 \cdot \mathbf{p}_1) + 2(\boldsymbol{\sigma}_2 \cdot \mathbf{p}_1)(\boldsymbol{\sigma}_1 \cdot \mathbf{q}) + 2(\boldsymbol{\sigma}_2 \cdot \mathbf{q})(\boldsymbol{\sigma}_1 \cdot \mathbf{p}_1) \\ &\quad - (\boldsymbol{\sigma}_1 \cdot \mathbf{q})(\boldsymbol{\sigma}_2 \cdot \mathbf{q}) - 2(\boldsymbol{\sigma}_2 \cdot \mathbf{q})(\boldsymbol{\sigma}_2 \cdot \mathbf{p}_1) - 2(\boldsymbol{\sigma}_1 \cdot \mathbf{p}_1)(\boldsymbol{\sigma}_1 \cdot \mathbf{q})\Big]\left(\frac{\Lambda^2 - m_\pi^2}{\Lambda^2 + \mathbf{q}^2}\right)^2, \end{aligned} \tag{5}$$

where m is the mass of nucleon. Taking the Fourier transformation, one has the potential caused by the single-pion-exchange in the configuration space as

$$\begin{aligned} V_\pi(r) &= \frac{g^2}{4m^2}\Big[4\mathbf{p}_1^2 f_\pi(r) - 4(\boldsymbol{\sigma}_1 \cdot \mathbf{p}_1)(\boldsymbol{\sigma}_2 \cdot \mathbf{p}_1)f_\pi(r) - 2i(\boldsymbol{\sigma}_2 \cdot \mathbf{p}_1)(\boldsymbol{\sigma}_1 \cdot \mathbf{r})F_\pi(r) \\ &\quad - 2iF_\pi(r)(\boldsymbol{\sigma}_2 \cdot \mathbf{r})(\boldsymbol{\sigma}_1 \cdot \mathbf{p}_1) + (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\nabla})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{\nabla})f_\pi(r) + 2iF_\pi(r)(\boldsymbol{\sigma}_2 \cdot \mathbf{r})(\boldsymbol{\sigma}_2 \cdot \mathbf{p}_1) \\ &\quad + 2i(\boldsymbol{\sigma}_1 \cdot \mathbf{p}_1)(\boldsymbol{\sigma}_1 \cdot \mathbf{r})F_\pi(r)\Big]. \end{aligned} \tag{6}$$

This potential is not hermitian, we need to re-order the operator $\mathbf{p}$ and the functions of r and the standard treatment is the Weyl ordering procedure [11]. The resultant effective potential is

$$\begin{aligned} &V_\pi(r)_{Weyl} = \\ &\quad \frac{g^2}{4m^2}\Big\{[f_\pi(r)\mathbf{p}_1^2 + \mathbf{p}_1 f_\pi(r)\mathbf{p}_1 + \mathbf{p}_1^2 f_\pi(r)] - [f_\pi(r)(\boldsymbol{\sigma}_1 \cdot \mathbf{p}_1)(\boldsymbol{\sigma}_2 \cdot \mathbf{p}_1) \\ &\quad + (\boldsymbol{\sigma}_1 \cdot \mathbf{p}_1)f_\pi(r)(\boldsymbol{\sigma}_2 \cdot \mathbf{p}_1) + (\boldsymbol{\sigma}_1 \cdot \mathbf{p}_1)(\boldsymbol{\sigma}_2 \cdot \mathbf{p}_1)f_\pi(r)] - i[(\boldsymbol{\sigma}_2 \cdot \mathbf{p}_1)(\boldsymbol{\sigma}_1 \cdot \mathbf{r})F_\pi(r) \\ &\quad + F_\pi(r)(\boldsymbol{\sigma}_1 \cdot \mathbf{r})(\boldsymbol{\sigma}_2 \cdot \mathbf{p}_1)] - i[(\boldsymbol{\sigma}_1 \cdot \mathbf{p}_1)(\boldsymbol{\sigma}_2 \cdot \mathbf{r})F_\pi(r) + F_\pi(r)(\boldsymbol{\sigma}_2 \cdot \mathbf{r})(\boldsymbol{\sigma}_1 \cdot \mathbf{p}_1)] \\ &\quad + i[(\boldsymbol{\sigma}_1 \cdot \mathbf{p}_1)(\boldsymbol{\sigma}_1 \cdot \mathbf{r})F_\pi(r) + F_\pi(r)(\boldsymbol{\sigma}_1 \cdot \mathbf{r})(\boldsymbol{\sigma}_1 \cdot \mathbf{p}_1)] + i[(\boldsymbol{\sigma}_2 \cdot \mathbf{p}_1)(\boldsymbol{\sigma}_2 \cdot \mathbf{r})F_\pi(r) \\ &\quad + F_\pi(r)(\boldsymbol{\sigma}_2 \cdot \mathbf{r})(\boldsymbol{\sigma}_2 \cdot \mathbf{p}_1)] + [(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\nabla})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{\nabla})f_\pi(r)]\Big\}, \end{aligned} \tag{7}$$

where

$$\begin{aligned} f_\pi(r) &= \frac{e^{-m_\pi r}}{4\pi r} - \frac{e^{-\Lambda r}}{4\pi r} + \frac{(m_\pi^2-\Lambda^2)e^{-\Lambda r}}{8\pi\Lambda} \\ F_\pi(r) &= \frac{1}{r}\frac{\partial}{\partial r} f_\pi(r). \end{aligned} \tag{8}$$

And it is the form we are going to use in the later part of the work.

(b) Via σ and ρ and ω exchanges.
The effective vertices are respectively

$$L_\sigma = g\bar{\psi}\psi\sigma, \tag{9}$$

$$L_\rho = g_{NN\rho}\bar{\psi}\gamma_\mu\tau^a\psi A^{a\mu} \quad a = 1,2,3, \tag{10}$$

and for $\omega-$vector meson it is

$$L_\omega = g_{NN\omega}\bar{\psi}\gamma_\mu\psi\omega^\mu. \tag{11}$$

Neglecting some technical details, one can easily derive the corresponding effective potentials as

$$\begin{aligned} V_\sigma(r) &= g^2\Big\{ -f_\sigma(r) + \frac{[f_\sigma(r)\mathbf{p}_1^2 + \mathbf{p}_1 f_\sigma(r)\mathbf{p}_1 + \mathbf{p}_1^2 f_\sigma(r)]}{4m^2} \\ &- \frac{[\boldsymbol{\nabla}^2 f_\sigma(r)]}{4m^2} + \frac{i[\mathbf{p}_1\cdot\mathbf{r}F_\sigma(r) + F_\sigma(r)\mathbf{r}\cdot\mathbf{p}_1]}{2m^2} - \frac{(\mathbf{L}\cdot\mathbf{S})F_\sigma(r)}{2m^2}\Big\}, \end{aligned} \tag{12}$$

where

$$\begin{aligned} f_\sigma(r) &= \frac{e^{-m_\sigma r}}{4\pi r} - \frac{e^{-\Lambda r}}{4\pi r} + \frac{(m_\sigma^2-\Lambda^2)e^{-\Lambda r}}{8\pi\Lambda} \\ F_\sigma(r) &= \frac{1}{r}\frac{\partial}{\partial r} f_\sigma(r). \end{aligned} \tag{13}$$

and

$$\begin{aligned} V_\rho(r) &= \\ &\frac{g_{NN\rho}^2}{4m^2}\Big\{4m^2 f_\rho(r) + [\boldsymbol{\nabla}^2 f_\rho(r)] - \frac{[f_\rho(r)\mathbf{p}_1^2 + \mathbf{p}_1 f_\rho(r)\mathbf{p}_1 + \mathbf{p}_1^2 f_\rho(r)]}{2} + 2F_\rho(r)\mathbf{L}\cdot\mathbf{S} \\ &- i[\mathbf{p}_1\cdot\mathbf{r}F_\rho(r) + F_\rho(r)\mathbf{r}\cdot\mathbf{p}_1] + \frac{5}{2}[f_\rho(r)(\boldsymbol{\sigma}_1\cdot\mathbf{p}_1)(\boldsymbol{\sigma}_2\cdot\mathbf{p}_1) + (\boldsymbol{\sigma}_1\cdot\mathbf{p}_1)f_\rho(r)(\boldsymbol{\sigma}_2\cdot\mathbf{p}_1) \\ &+ (\boldsymbol{\sigma}_1\cdot\mathbf{p}_1)(\boldsymbol{\sigma}_2\cdot\mathbf{p}_1)f_\rho(r)] + \frac{5i}{2}[F_\rho(r)(\boldsymbol{\sigma}_1\cdot\mathbf{r})(\boldsymbol{\sigma}_2\cdot\mathbf{p}_1) + (\boldsymbol{\sigma}_2\cdot\mathbf{p}_1)(\boldsymbol{\sigma}_1\cdot\mathbf{r})F_\rho(r)] \\ &+ \frac{5i}{2}[F_\rho(r)(\boldsymbol{\sigma}_2\cdot\mathbf{r})(\boldsymbol{\sigma}_1\cdot\mathbf{p}_1) + (\boldsymbol{\sigma}_1\cdot\mathbf{p}_1)(\boldsymbol{\sigma}_2\cdot\mathbf{r})F_\rho(r)] - [(\boldsymbol{\sigma}_1\cdot\boldsymbol{\nabla})(\boldsymbol{\sigma}_2\cdot\boldsymbol{\nabla})f_\rho(r)] \\ &+ \frac{1}{2}[(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)\cdot\mathbf{p}_1(\boldsymbol{\sigma}_1\cdot\mathbf{r})F_\rho(r) + F_\rho(r)(\boldsymbol{\sigma}_1\cdot\mathbf{r})(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)\cdot\mathbf{p}_1] \\ &+ \frac{1}{2}[F_\rho(r)(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)\cdot\mathbf{r}(\boldsymbol{\sigma}_2\cdot\mathbf{p}_1) + (\boldsymbol{\sigma}_2\cdot\mathbf{p}_1)(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)\cdot\mathbf{r}F_\rho(r)] \end{aligned}$$

$$
-\frac{1}{2}[F_\rho(r)(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)\cdot\mathbf{r}(\boldsymbol{\sigma}_1\cdot\mathbf{p}_1)+(\boldsymbol{\sigma}_1\cdot\mathbf{p}_1)(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)\cdot\mathbf{r}F_\rho(r)]
$$

$$
-\frac{1}{2}[(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)\cdot\mathbf{p}_1(\boldsymbol{\sigma}_2\cdot\mathbf{r})F_\rho(r)+F_\rho(r)(\boldsymbol{\sigma}_2\cdot\mathbf{r})(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)\cdot\mathbf{p}_1]\Big\}, \tag{14}
$$

where

$$
\begin{aligned}
f_\rho(r) &= \frac{e^{-m_\rho r}}{4\pi r}-\frac{e^{-\Lambda r}}{4\pi r}+\frac{(m_\rho^2-\Lambda^2)e^{-\Lambda r}}{8\pi\Lambda}\\
F_\rho(r) &= \frac{1}{r}\frac{\partial}{\partial r}f_\rho(r).
\end{aligned} \tag{15}
$$

The potential induced by exchanging ω vector meson is in close analogy to V_ρ, with only m_ρ being replaced by m_ω. We neglect the contributions by exchanging heavier mesons.

The total effective potential is the sum of these potentials induced respectively via exchanging π, σ and ρ, ω mesons.

$$
V_{eff}(r)=V_\pi(r)+V_\sigma(r)+V_\rho(r)+V_\omega(r).
$$

After simple manipulations, we can write the $V_{eff}(r)$ as a sum containing a central potential term $V_C(r)$, a spin-dependent central potential part $V_S(r)\ \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$, a spin-orbit coupling part $V_{LS}(r)\ \mathbf{L}\cdot\mathbf{S}$, and a tensor interaction part $V_T(r)\ S_{12}$. The tensor operator $S_{12}=[3(\boldsymbol{\sigma}_1\cdot\hat{\mathbf{r}})(\boldsymbol{\sigma}_2\cdot\hat{\mathbf{r}})-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2]$ is known to admix the s-wave and d-wave states, so we separate out the tensor part of the potential as

$$
\begin{aligned}
V_T(r)S_{12} = &\Bigg[\Bigg(-\frac{e^{-\Lambda r}\Lambda^3 g_{NN\rho}^2}{64m^2\pi}+\frac{e^{-\Lambda r}\Lambda m_\rho^2 g_{NN\rho}^2}{64m^2\pi}-\frac{3e^{-\Lambda r}g_{NN\rho}^2}{32m^2\pi r^3}+\frac{3e^{-m_\rho r}g_{NN\rho}^2}{32m^2\pi r^3}-\frac{3e^{-\Lambda r}\Lambda g_{NN\rho}^2}{32m^2\pi r^2}\\
&+\frac{3e^{-m_\rho r}m_\rho g_{NN\rho}^2}{32m^2\pi r^2}-\frac{3e^{-\Lambda r}\Lambda^2 g_{NN\rho}^2}{64m^2\pi r}+\frac{e^{-\Lambda r}m_\rho^2 g_{NN\rho}^2}{64m^2\pi r}+\frac{e^{-m_\rho r}m_\rho^2 g_{NN\rho}^2}{32m^2\pi r}\Bigg)\\
&+\Bigg(-\frac{e^{-\Lambda r}\Lambda^3 g_{NN\omega}^2}{64m^2\pi}+\frac{e^{-\Lambda r}\Lambda m_\omega^2 g_{NN\omega}^2}{64m^2\pi}-\frac{3e^{-\Lambda r}g_{NN\omega}^2}{32m^2\pi r^3}+\frac{3e^{-m_\omega r}g_{NN\omega}^2}{32m^2\pi r^3}\\
&-\frac{3e^{-\Lambda r}\Lambda g_{NN\omega}^2}{32m^2\pi r^2}+\frac{3e^{-m_\omega r}m_\omega g_{NN\omega}^2}{32m^2\pi r^2}-\frac{3e^{-\Lambda r}\Lambda^2 g_{NN\omega}^2}{64m^2\pi r}+\frac{e^{-\Lambda r}m_\omega^2 g_{NN\omega}^2}{64m^2\pi r}\\
&+\frac{e^{-m_\omega r}m_\omega^2 g_{NN\omega}^2}{32m^2\pi r}\Bigg)\Bigg]S_{12},
\end{aligned} \tag{16}
$$

and treat it as a perturbation later.

Now, the Schrödinger equations for s-wave state (^{3}S$_1$) and d-wave state (^{3}D$_1$) have the following forms

$$
\left[\frac{\mathbf{p}^2}{2\mu}+V_C(r)+V_S(r)\right]\Psi_s(\mathbf{r})=E_s\Psi_s(\mathbf{r}), \tag{17}
$$

$$
\left[\frac{\mathbf{p}^2}{2\mu}+V_C(r)+V_S(r)-3V_{LS}(r)-2V_T(r)\right]\Psi_d(\mathbf{r})=E_d\Psi_d(\mathbf{r}), \tag{18}
$$

where μ is the reduced mass, here it is $\frac{1}{2}M_p=\frac{1}{2}M_n$. Solving these equations, we obtain the eigen-energies and eigen-wavefunctions of the s-wave and d-wave states. Then the tensor part, which results in a mixing fraction between the s- and d-waves, would be taken in as a perturbation.

We have a modified hamiltonian as

$$H_{full} = \begin{pmatrix} E_s & \Delta \\ \Delta^* & E_d \end{pmatrix}. \tag{19}$$

Here E_s and E_d are the eigenvalues of s- and d-waves obtained by solving the Schrödinger equations (17) and (18), Δ is defined as

$$\Delta = < s|V_T(r)S_{12}|d > .$$

Diagonalizing H_{full}, we obtain the real eigenvalues and identify the lower one to be the binding energy of deuteron (we will discuss this issue in the last section again), at the same time we can determine the fraction of the d-wave in the eigenfunction

$$|\Psi >= c_1|s> + c_2|d> .$$

III. The numerical results

The input parameters include the masses $m_\pi = 0.139$ GeV, $m_\rho = 0.77$ GeV, $m_\omega = 0.783$ GeV, $m_p \approx m_n = 0.938$ GeV, and the couplings $g_{_{NN\sigma}} = g_{_{NN\pi}} = 13.5$, $g_{_{NN\rho}} = 3.25$ [12]. In the exact SU(3) limit, by a simple quark counting, $g_{_{NN\omega}} = 3g_{_{NN\rho}}$, but our later numerical results show that this relation does not lead to a satisfactory solution. For a comparison, we will take three different $g_{_{NN\omega}}$ values which are $g_{_{NN\rho}}$, $2g_{_{NN\rho}}$ and $3g_{_{NN\rho}}$ respectively and present the results in tables 1,2 and 3.

Even though we do not have direct information about $g_{NN\sigma}$, $g_{NN\sigma} = g_{NN\pi}$ is imposed by the linear sigma model. Λ is a free parameter in the form factor, it is generally believed that its value is in a range around 1 GeV. Therefore, one can vary it within a reasonable region around 1 GeV. Here we take it as a free parameter close to 1 GeV and adjust it to fit the data of deuteron. When we carry out the numerical computations, by imposing the experimental data as constraints which include the binding energy of deuteron, a mixing between s- and d-waves and the charge radius etc., it is found that a unique $\Lambda-$value for both s- and d-waves cannot lead to a reasonable solution set which can meet the data. Therefore we take separate values for s- and d-waves, more discussions will be given in the last section.

The main task of this work is to find the mass of $\sigma-$particle, it stands as free a parameter in the present calculation and we will scan the possible region and finally determine its value by fitting data. As well known, $\sigma-$meson is a rather wide resonance, it is so far not well experimentally determined yet, the data allow a large range as the mass being $400 \sim 1200$ MeV and width being $600 \sim 1000$ MeV[2]. For deriving the effective potential the width is irrelevant, because the t-channel is a deep space-like region, so the width is not involved, thus in this work we cannot determine the width from the static properties of deuteron. If one wishes to study the width, he must calculate the production rate or decay (lifetime) of deuteron which would be more difficult and we will deal with it in our later work.

The experimental value of the binding energy of deuteron is [15]

$$E_b^{exp} = -2.224575 \text{ MeV}.$$

Our strategy is to fit the binding energy which is very accurately measured and serves as a rigorous criterion for our computation. Then we achieve a reasonable mixing fraction between

the s- and d-waves which has also been experimentally confirmed. As aforementioned, to fulfil all the criteria, a unique $\Lambda-$ value is not enough, thus we set Λ_s and Λ_d for s- and d-waves respectively when we solve the Schrödinger equations. We present our numerical results in Table 1.

m_σ ($\times 10^{-1}$GeV)	Λ_s (GeV)	Λ_d (GeV)	E_S (MeV)	E_D (MeV)	Δ (MeV)	$(\frac{C_d}{C_s})^2$	$\bar{r}$ (fm)
4.70	0.59	1.35	-1.97	-0.29	0.71	0.13	3.54
4.80	0.60	1.40	-2.00	-0.36	0.64	0.12	3.47
4.90	0.61	1.46	-2.04	-0.40	0.58	0.10	3.39
5.00	0.62	1.52	-2.08	-0.43	0.52	0.08	3.31
5.10	0.63	1.58	-2.11	-0.44	0.46	0.07	3.23
5.20	0.64	1.65	-2.13	-0.48	0.40	0.05	3.16
5.30	0.65	1.71	-2.15	-0.51	0.35	0.04	3.08
5.40	0.66	1.78	-2.17	-0.54	0.29	0.03	3.01
5.50	0.67	1.84	-2.19	-0.55	0.25	0.02	2.94
5.60	0.69	1.91	-2.22	-0.58	0.20	0.01	2.88

Table 1. $g_{NN\pi} = g_{NN\sigma} = 13.5, g_{NN\rho} = 3.25, g_{NN\omega} = g_{NN\rho}$ All the other parameters are obtained by fitting the binding energy $E_b^{exp} = -2.22$ GeV

With another set of couplings, we have

m_σ ($\times 10^{-1}$GeV)	Λ_s (GeV)	Λ_d (GeV)	E_S (MeV)	E_D (MeV)	Δ (MeV)	$(\frac{C_d}{C_s})^2$	$\bar{r}$ (fm)
5.10	0.64	1.75	-1.70	-0.30	1.00	0.27	3.54
5.20	0.65	1.84	-1.80	-0.33	0.90	0.22	3.45
5.30	0.66	1.92	-1.89	-0.36	0.79	0.18	3.36
5.40	0.67	2.02	-1.97	-0.40	0.68	0.14	3.28
5.50	0.68	2.11	-2.04	-0.43	0.58	0.10	3.19
5.60	0.69	2.21	-2.09	-0.46	0.48	0.08	3.10
5.70	0.70	2.30	-2.13	-0.50	0.39	0.05	3.02
5.80	0.71	2.40	-2.17	-0.51	0.31	0.03	2.92

Table 2. $g_{NN\pi} = g_{NN\sigma} = 13.5, g_{NN\rho} = 3.25, g_{NN\omega} = 2g_{NN\rho}$ All the other parameters are obtained by fitting the binding energy $E_b^{exp} = -2.22$ GeV

Then for a comparison we give another set of results which is listed in table 3.

m_σ ($\times 10^{-1}$GeV)	Λ_s (GeV)	Λ_d (GeV)	E_S (MeV)	E_D (MeV)	Δ (MeV)	$(\frac{C_d}{C_s})^2$	$\bar{r}$ (fm)
3.50	0.53	0.79	-2.16	-0.44	3.81	0.64	4.45
4.00	0.57	1.03	-2.08	-0.40	3.46	0.62	3.97
4.50	0.61	1.46	-1.92	-0.25	2.72	0.55	3.67
5.00	0.65	2.14	-1.84	-0.19	1.86	0.43	3.44
5.50	0.69	3.07	-1.68	-0.09	1.02	0.24	3.36

Table 3. $g_{NN\pi} = g_{NN\sigma} = 13.5$,$g_{NN\rho} = 3.25$,$g_{NN\omega} = 3g_{NN\rho}$ All the other parameters are obtained by fitting the binding energy $E_b^{exp} = -2.22$ GeV.

Obviously, even though all the three sets of couplings can lead to results which coincide with the data on the binding energy, when we consider other experimental constraints, especially the s-wave and d-wave mixing fraction of about 4%, the set given in table 3 is not favored.

It is noted that to fit the binding energy of deuteron and consider the constraint of the s- and d-wave mixing, the estimated mass of σ meson deviates for different parameter sets by about 50 MeV. We will further discuss this issue in the last section. And we have also noted that for each parameter set, beyond the range of m_σ, we cannot find any reasonable solution for the binding energy and the mixing parameter $(C_d/C_s)^2$.

IV. Discussions and conclusion

Nowadays, the hadron structure is still a hot topic in particle physics, we wonder if the observed hadron spectra from light to heavy can be fully interpreted in the quark model. Among all the light hadrons, the scalar σ $(0^+,\ f_0(600))$ may take the most noticeable position. Dispute about its existence as a physical resonance has been on for a long time. A recent observation of the narrow resonance (2.32 GeV/c^2) by the BABAR collaboration [19] has triggered a new tide to discuss the whole family of the 0^+ particles[20] where σ is the leading one. One of the most striking questions is whether they can be attributed to the building of $q\bar{q}$ system or some of them may be molecular states. Indeed, the measured resonance peaks $f_0(980)$, $a_0(980)$ are interpreted as $K\bar{K}$ molecula [8], Rujula, Georgi and Glashow suggest that $\Psi(4.04)$ is a $D^*\bar{D}^*$ molecula [14] etc. To further investigate such molecular states, one needs to know the interaction between the meson-constituents in the molecular states and derive the effective potential. In that case, the σ particle may be the main agent to mediate the interaction as well as pion and other light vector mesons. Thus to test the properties of σ becomes the most important task in the whole study.

There are many ways to study σ, among them the structure of deuteron provides us with direct information about the σ particle. Deuteron is the simplest bound state of nucleons, in general, it is a "molecular" state of two fermions (proton and neutron). Because its spectrum is accurately measured, it is an ideal probe for the applicable mechanism and theoretical framework.

In this work, we employ the linear σ model where the σ is a real physical particle whereas in the chiral Lagrangian approach its role is replaced by the two-pion-exchange picture. At the effective vertices, we retain a form factor with a free parameter Λ which is determined by fitting data.

When we consider the constraint from the experimental observation that a certain fraction of about 4% d-wave mixes with the s-wave, we fit the binding energy and the results are listed in Table 1. As aforementioned, a unique Λ value cannot lead to a solution for the binding energy and mixing of around 4%. Thus we take two parameters Λ_s and Λ_d corresponding to s- and d-waves respectively. In fact, Λ represents the inner structure of the constituents of deuteron and the effective interaction between them. For s- and d-waves, the orbital angular momenta are different, and the momentum distributions of the constituents in the s- and d-states behave differently, namely the effective binding could deviate from each other. The different $\Lambda-$ values for s- and d-waves would manifest the difference in effective interaction for different angular momenta states.

The effective coupling $g_{_{NN\sigma}} = g_{_{NN\pi}} = 13.5$ is taken in our numerical computations, but other possibilities are also noted that in literature, a different value of the coupling is also commonly used as $g_{_{NN\sigma}} = g_{_{NN\pi}} = 10.1$[16]. To investigate the influence of different couplings, we repeat all the numerical computations with the coupling being replace by $g_{_{NN\sigma}} = g_{_{NN\pi}} = 10.1$, the results are not in a good shape, namely no reasonable solutions for both the binding energy and mixing fraction can be reached. Thus this model does not advocate smaller coupling $g_{_{NN\sigma}} = g_{_{NN\pi}} = 10.1$.

It is worth noticing that the results somewhat depend on the coupling $g_{_{NN\omega}}$. Indeed, both the ρ and ω mesons provide an effective short-range expelling interaction between nucleons, by the SU(3) symmetry, $g_{_{NN\phi}} = 0$ and $g_{_{NN\omega}} \approx 3g_{_{NN\rho}}$ [12], if a simple quark counting rule is used. But the relation does not need to be taken seriously, because the SU(3) symmetry is obviously broken. In fact, in the literature, quite different relations between $g_{_{NN\omega}}$ and $g_{_{NN\rho}}$ have been adopted. One would rather use the loose relation as $g_{_{NN\omega}} = \alpha g_{_{NN\rho}}$ where α is treated as a free parameter and is to be determined by fitting data. In the text, we adopt three different α values to be 1, 2 and 3 respectively and present corresponding results in the tables. We observe that all the results corresponding to the three relations can lead to solutions which perfectly coincide with the data of the deuteron binding energy, but while considering the s-wave and d-wave mixing fraction $|c_d/c_s|^2$ as a constraint, one can immediately notice that for larger α values the fraction becomes unbearably large. It indicates that smaller α values are more favored, and it may also manifest the SU(3) breaking in this case or in other words, for an effective theory for the NN interaction at very low energy (near zero), $g_{_{NN\omega}} \approx g_{_{NN\rho}}$.

As a conclusion of this work, the data on deuteron advocates the mass range of the σ particle around 520 to 580 MeV which is consistent with the present data and the value commonly used in the phenomenological nuclear physics. Meanwhile we obtain a fraction of the d-wave in the total wave function as about 4$\sim$ 10% and it also coincides with the observation. One question might be raised that if deuteron is a mixed state of the s- and d-waves with the lower eigen-energy, where is the bound state of the higher energy. Since deuteron is the unique stable nucleus consisting of a proton and a neutron, no other bound state has been experimentally observed, it implies that the bound state of higher eigen-energy is unstable and easy to dissociate, so does not exist in the nature. Indeed, our calculation shows that as the binding energy of deuteron is -2.22 MeV, another eigen-energy is very close to zero.

It has been reported that the newly measured value of the mass and width of $\sigma-$meson as

$$m_\sigma = 390^{+60}_{-36}\ \text{MeV} \quad \text{and} \quad \Gamma_\sigma = 282^{+77}_{-50}\ \text{MeV[17]}.$$

Huo et al. recently also discussed on the σ properties in J/ψ decays [18]. Their value of the mass of $\sigma-$meson might be a bit smaller than the generally expected. Instead, in this work we use the deuteron data to investigate the mass. Because of the measurement errors, we take several m_σ values and recalculate E_b.

In general, our result indicates that the linear σ model may apply to determine phenomenological quantities of deuteron and inversely by studying these quantities, one can achieve further information about this long-expected particle $f_0(600)$. As many theorists and experimentalists concern its existence and search for it at various experimental facilities and processes, we believe that its existence will be confirmed soon and its properties including mass and lifetime will be fixed with much higher accuracy in near future. Then we will be able to further study our mechanism and the parameter Λ. These results can be compared with other data of wider energy ranges for getting a better insight into the strong interactions. That achievements would enrich our knowledge about the interaction mediated by the σ meson and then we will apply it to other areas of high energy physics and nuclear physics. However, from the other side, we find that the results depend on the adopted parameters which are obtained by fitting the data of nucleon scattering and given in different works of literature. Further study on σ meson would provide more definite information about its mass and then we can continue to investigate the linear σ model.

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